Stochastic valuation of futures contracts of IPC in the Mexican derivatives market

ORTEGA- Miguel†*& GALICIA-Alexander

Instituto Politécnico Nacional. Av Juan de Dios Bátiz S/N, Gustavo A. Madero, Residencial La Escalera, 07738 Ciudad de México, Distrito Federal, México

This paper empirically evaluates the stochastic behavior of the price of futures contracts of IPC which are traded in the derivatives market in the second quarter of 2011; It is assumed that the process of dissemination of the price is represented by the model of the geometric Brownian motion and measured with the process of random walk, which occurs during the life of the contract up to the expiration date. The evaluation is carried out through a process of Monte Carlo simulation that allows to analyze all possibilities of the behavior of the evolution of the indicator of the index of prices and exchange rates (IPC) and based on this information determines the price of futures whose underlying asset is the IPC, for the purposes of calculating the performance of the index is used and is a log-normal representation of the value of the index that is more realistic because they do not permit values to zero. Empirical evidence shows that the stochastic model of the geometric Brownian motion is actually a good predictor that models the behavior of the price of futures contracts during its useful life, which is before its expiry date.

Future contract, active index, underlying price and contract quotes, stochastic process, geometric Brownian motion, random walk

---

*Correspondence to Author (email: miflores@ipn.mx)

† Researcher contributing first author.

Introduction

In the real world dynamics of the economic environment is reflected in the representative market index values; a way to represent is by means of a stochastic process that describes the diffusion of the index, so in this research to determine the parameters of the geometric Brownian model is analyzed to represent the behavior of the index, and determine the price of futures contracts on it, compared to historical records observed in the Mexican derivatives market (MexDer).

This paper presents the empirical testing of random walk stochastic model and compared with the model of geometric Brownian motion, for which the performance of the price index (IPC) and the pricing of the simulated futures contracts that utilizes the underlying assets.

The effect of the time interval is analysed assessment in the price of futures contracts, when are compared with the spread of the index of prices and rates as the expiration date of the futures contract approaches. For copying behavior initially adjusted yields for a log-normal representation of the index where values below zero are not allowed.

Additionally, the results of 53 simulations are used to show the stochastic behavior of underlying asset value and confirm the study hypothesis.

Reporting relationships convergence between the futures price and the market price at the approach of the expiration date of the futures contract that is traded on the MexDer.

The paper is organized into six sections; The first introduces the problem, in the second section the model is used to represent the stochastic process, the third is presented the development of an empirical model to evaluate futures contracts in June 2011 CPI series, the fourth section presents analyzed data in the fifth section the results are presented, and the sixth section concludes with suggestibility.

A stochastic process model

In uncertain environment, if, $S(t)$ represents the resulting value of $ 1 invested with compound interest and continuous compounding for a constant rate μ during the period $[0, t]$, then $S(t)$ is the solution of the problem with an initial of $S(0)$, with an ordinary differential equation which indicates that capital grows at a constant rate and equal to μ, with an initial condition that indicates that the investment at the beginning of the next interval is:

$$\frac{dS(t)}{dt} = \mu S(t), \quad S(0) = 1 \quad (1)$$

When the investment is made on a stock exchange, is more realistic to consider that the rate of growth of an investment contains uncertainty, and is usually said to be a normal stochastic process with zero mean and variance t, which is called Brownian motion, and S (t) is the derivative of the equation, that according to the conventional theory, the trajectories of the stochastic process are not differentiable at any point.

If $\dot{B}(t)$ is a stochastic process and stationary called white noise.

In this context, the difference given in (1) is written in the form:

$$\frac{dS(t)}{dt} = \left(\mu + \sigma \dot{B}(t)\right) S(t), \qquad (2)$$

or in its differential representation:

$$dS(t) = \left(\mu S(t)dt + \sigma S(t)\right)dB(t) \qquad (3)$$

This expression is formal and is called stochastic differential equation. When $\sigma = 0$, corresponds to a deterministic or without model uncertainty with a solution for the initial condition $S(0) = 1$, which is expressed by:

$$S(t) = \exp(\mu t), \qquad (4)$$

But if $\sigma \neq 0$, the solution requires to differentiate the stochastic process, and has no conventional mathematical tools for the solution, so is necessary to use stochastic calculus of Itô or calculation, which is a tool to work with stochastic differential equations and obtain a solution of the form:

$$X(t) = X_0 + \int_0^t A_1(s)ds + c\int_0^t A_2(s)dB(s) \qquad (5)$$

Where:

$A_1(s)$ and $A_2(s)$ are stochastics process adapted to $\sigma(B(s) : s \leq t)$ is the σ- algebra generated by $B(s)$ with $s \leq t$.

In (5), the first integral is an ordinary integral and the second is an Itô stochastic integral. The hypothesis can be substantially relax assuming that A1 (s) and A2 (s) are continuous functions whose integral corresponds to unity probability, and it holds that:

$$\int_0^t A_2(s))^2 ds, \qquad (6)$$

To $< +\infty$ and therefore states that the integral of the equation (5) are defined.

Deduction of geometric Brownian motion

The main tool of development Itô stochastic calculus that plays the role analogous to the chain rule, and is given through the result known as Itô slogan.

Given a stochastic differential equation of the form (5) and f (t, x) a continuous function with a first-order partial derivative with respect to t continuous and second-order derivative with respect to X, for S <t, holds:

$$f\big(t,B(t)\big) - f\big(s,B(s)\big) = \int_s^t \left(\frac{\partial f(u,B(u))}{\partial t} + \frac{1}{2}\frac{\partial^2 f(u,B(u))}{\partial x^2}\right)du\} \\ + \int_s^t \frac{\partial f(u,B(u))dB}{\partial x} dB(u) \qquad (7)$$

For the application of this result, first write the stochastic differential equation (3) with initial condition S (0) = S0 as an integral:

$$(t) = S_0 + \int_0^t \mu S(x)dx + \int_0^t \sigma S(x)dB(x), \qquad (8)$$

And it is supposed that $S(t) = f(t, B(t))$, then identifying quotients in (7) and (8) we have:

$$\mu f(t, x) = \frac{\partial f(t,x)}{\partial t} + \frac{1}{2}\frac{\partial^2 f(t,x)}{\partial x^2}, \qquad (9)$$

$$\sigma f(t, x) = \frac{\partial f(t,x)}{\partial x}, \qquad (10)$$

Differentiating (10) with respect to the variable, stochastic integral, x, is obtained:

Ortega M. Galicia A. Stochastic valuation of futures contracts of IPC in the Mexican derivatives market. ECORFAN Journal-Mexico 2011, 2-5: 380-394

$$\sigma \frac{\partial f(t,x)}{\partial t} = \frac{\partial^2 f(t,x)}{\partial x^2}, \qquad (11)$$

And substituting (10) into (11) gives:

$$\sigma^2 f(t,x) = \frac{\partial^2 f(t,x)}{\partial x^2} \qquad (12)$$

Simplifying (9) and (12), the partial derivatives are obtained:

$$\left(\mu - \frac{1}{2}\sigma^2\right) f(t,x) = \frac{\partial f(t,x)}{\partial t} \; ; \; \sigma f(t,x) = \frac{\partial f(t,x)}{\partial t}, \qquad (13)$$

The solution is sought, using the method of separation of variables for partial differential equations in the form:

$$f(t,x) = g(t)h(x), \qquad (14)$$

With according to (13) must be satisfied:

$$g(t) = g(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t\right); h(x) = h(0)\exp(\sigma x), \qquad (15)$$

Thus,

$$f(t,x) = g(t)h(x) = g(0)h(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma x\right), \qquad (16)$$

and by definition of Brownian motion B (0) = 1, continuous unit probability, we have:

$$S(0) = f\big(0, B(0)\big) = f(0,0) = g(0)h(0), \qquad (17)$$

Then:

$$f(t,x) = S(0)\exp\left(\left(\hat{u} - \frac{\sigma^2}{2}\right)t + \sigma\right) \qquad (18)$$

Finally,

$$S(t) = f\big(t, B(t)\big) = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B(t)\right). \qquad (19)$$

Is stochastic process solution for $t \geq 0$ of the stochastic differential equation (3), referred to the geometric Brownian motion or literature log-normal stochastic process since for each t is the exponential of a random variable B (t).

Geometric Brownian motion properties

Brownian motion in a continuous process that has the following properties:

- $W_0 = 0$.

- For all $t \geq 0$, $W_t \sim N\,(0,t)$,i.e, W_t is a normally distributed variable with mean 0 and variance t.

- All the increases $\Delta W_t = W_t + \Delta_t - W_t$ are independent, i.e, for all $0 \leq t_1 < t_2 \leq t_3 < t_4$ the displacements $W_{t2} - W_{t1}$ y $W_{t4} - W_{t3}$ are independent.

- W_t depends on t.

The algorithm for the numerical simulation of Brownian motion is as follows:

From initial values:

$$W_0 = 0, t_0 = 0, \Delta t, \qquad (20)$$

$$\text{para } j = 1, 2, \dots$$

$$t_j = t_j - 1 + \Delta t \qquad (21)$$

$$Z \sim N(0,1) \qquad (22)$$

$$W_j = W_{j-1} + Z\sqrt{\Delta t} \qquad (23)$$

Ortega M. Galicia A. Stochastic valuation of futures contracts of IPC in the Mexican derivatives market. ECORFAN Journal-Mexico 2011, 2-5: 380-394

Where Z is a random number normally distributed with zero mean and unit variance. Brownian motion is presented in Graphic 1.

Trajectories of Brownian motion to the daily prices of quarterly CPI 2010.
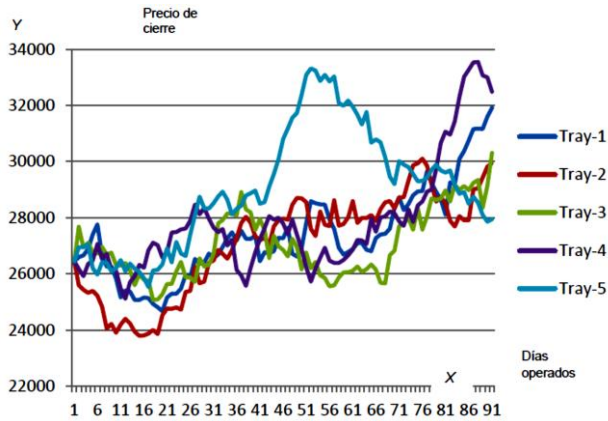
☒**Graphic 1**

In the geometric Brownian motion model of the lognormal hypothesis is imposed according to which, if S (t) is the random variable representing the price of the underlying asset at time t and initially is S_0, then:

$$\ln\left(\frac{S(t)}{S_0}\right) \sim \mathcal{N}\left(\left(\mu - \frac{1}{2}\sigma\right)t, \sigma^2\right) \tag{24}$$

Where:

- μ is the expected return on equities.

- A Hope of the random variable S (t) is:

$$\mathbb{E}(S(t)) = S_0 e^{\mu t} \tag{25}$$

Where:

- σ is the volatility of the stock price.

Uncertainty about future movements corresponds to the variance of S (t) that is given by:

$$var(S(t)) = S_0^2 e^{2\mu t}\left(e^{\sigma^2} - 1\right) \tag{26}$$

Based on these arguments, the share price at time t will be:

$$S(t) = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}Z}, \tag{27}$$

Where:

$$Z \sim \mathcal{N}(0,1)$$

Stochastic Brownian behavior model of prices of a financial asset

The geometric Brownian motion model, is a mathematical model describing the relationship between the current price of an asset and its possible future prices. The geometric Brownian motion model, states that future payments of an asset are normally distributed and that the standard deviation or volatility corresponds to a distribution that can be estimated with past data.

If the rate of payment of an asset between the present and future a brief moment, Dt is normally distributed. The mean of this distribution is μΔt and standard deviation σ √ (At). Technically, we assume that the price process S corresponds to the solution of the stochastic differential equation:

$$S_t = \mu S_t dt + \sigma S_t dB_t, \tag{28}$$

Ortega M. Galicia A. Stochastic valuation of futures contracts of IPC in the Mexican derivatives market. ECORFAN Journal-Mexico 2011, 2-5: 380-394

Therefore, if the price of an asset represents the variable S, Sten price at time t follows a geometric Brownian motion instantaneous average μ and instant standard deviation σ, then the payment rate S between time t and any other time T is given by:

$$Y = \frac{1}{T-t}\ln\left(\frac{S_T}{S_t}\right) \tag{29}$$

The variable is normally distributed with mean $\left(\mu - \frac{\sigma^2}{2}\right)(T - t)$ and standard deviation $\sigma \sqrt{(T - t)}$. With the simulation proves the probability that the payment rate Y, is greater than a given percentage α:

$$\Pr(|Y| \geq \alpha) = F\left(\frac{-\alpha-\mu+\frac{\sigma^2}{2}}{\sigma}\right) + F\left(-\frac{\alpha-\mu+\frac{\sigma^2}{2}}{\sigma}\right) \tag{30}$$

Stochastic convergence

The concept of stochastic convergence unlike its conceptualization in the field of real numbers is not unique because it extends to the case of sequences of random variables with different possibilities of convergence and each carries different requirements on the elements of each sequence. Can be defined as a sequence of random to a countable infinite set of variables of that kind of elements.

If $\{X_n\}n \in \mathbb{N}$ a sequence of random variables defined on a fixed probability space $(\Omega, \mathcal{F}, \mathbb{P})$, such that:

$$E[X_n - X|^q] \to 0 \tag{31}$$

When $n \to \infty$, in which Venegas (2007), denoted as $X \xrightarrow{n \to} {}^qX$, where each X_n is a random variable.

An example of a sequence of random variables is repeated indefinitely and independently an experiment that some event **A** has a probability of **P**.

Xi is the b (p) associated with the i-th iteration, the set {Xn} such binomial is an example of a sequence of random variables which have the characteristic of being independent and identically distributed. From the above Xi can be defined as:

$$Y_n = X_1 + \cdots + X_n \sim B(n.p).\{Y_n\}, \tag{32}$$

A sequence such that its elements are distributed with the same type of distribution changes with n and are not independent.

Random walk

A diffusion process is a persistent long-term movement of a variable over time, so it corresponds to a time series where the values fluctuate around its trend. There are two kinds of tendencies, a deterministic which is a constant function which varies with time, and the other stochastic which means that its value is random and changes with the time.

In Graph 2 the constant trend and stochastic trend, where deviations from the straight deterministic trend are random with no stationary mean is shown, and also do not contribute to the development of long-term time series as it quickly removed, however in the case of the random component stochastic trend affects the course of long-term time series.

Deterministic versus stochastic trend
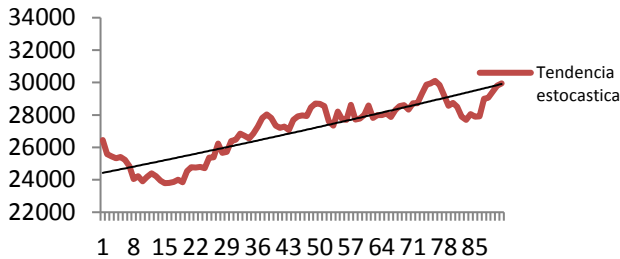
Graphic 2

Largely on the financial literature is said to represent the simplest movement that follow the values of financial assets is a random walk model. Therefore, a time series Y_t follows a random walk if the change in Yt corresponds to εt having a N (0,1) distribution represented by the following expression:

$$y_t = y_{t-1} + \varepsilon_t \qquad (33)$$

Where:

y_t - Logarithmic value of the asset over time.

y_{t-1} - Value of the asset and unpredictable change.

ε_t - Random error or change in value of the asset.

Referral that a time serie has a diffusion process if the random walk can be represented by a model based on this expression but with:

$$E[\varepsilon_t | y_{t-1}, y_{t-2}, \dots] = 0 \qquad (34)$$

Where:

E - Sample space of trajectories y_t.

y_{t-1}, y_{t-2} - Active steps in time.

$$E[\varepsilon_t | y_{t-1}, y_{t-2}, \dots] = y_{t-1} \qquad (35)$$

If y_t follows a random walk, the best predictor of tomorrow's value is the value today, so if series have a tendency to move carries an extension known as random walk with trend.

On the other hand if yt follows a random walk, the variance of the walk increases with time, so the distribution of yt also changes over time as follows:

$$y_t = y_{t-1} + \varepsilon_t, \qquad (36)$$

$$var(y_t) = var(y_{t-1}) + var(\varepsilon_t), \qquad (37)$$

$$var(y_t) \neq var(y_{t-1}) \qquad (38)$$

And it is said that the time series is not stationary and the trend shows that the distribution contains greater or equal than the unity variations.

Another way to look at the process yt is thinking that starts at zero, ie y0 = 0 then:

$$y_1 = \varepsilon_1, y_2 = \varepsilon_1 + \varepsilon_2, \qquad (39)$$

de tal forma que:

$$y_1 = \varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t \qquad (40)$$

Por lo tanto:

$$var(y_t) = var(\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_t) = t\sigma_\varepsilon^2 \qquad (41)$$

The populational autocorrelations of a random walk are not defined and the samples tend to unity. The random walk is a special case of an autoregressive model AR (1) with $\infty 1 = 0$, then yt has a stochastic trend and is not stationary.

If $|\infty_1| < 1$, then y_t is stationary. In the case of *AR(p)*, The condition that must be met to be stationary is that the solutions to the followed equation:

$$1 - \alpha_1 Z - \alpha_2 Z^2 - _{...} - \alpha_p Z^p = 0 \qquad (42)$$

Must be greater than the unity, so the polynomial roots must be outside the unit circle in the case of an AR (1), the result is:

$$z \frac{1}{\alpha_1}, \qquad (43)$$

So the root is greater than the unity in absolute value if $|\infty 1| < 1$.

Graph 3 represents a stationary and unchanged stochastic trend, as is normally distributed, ie, that εt follows a normal distribution with zero mean and constant variance less than the unity.

Random Walk without variations
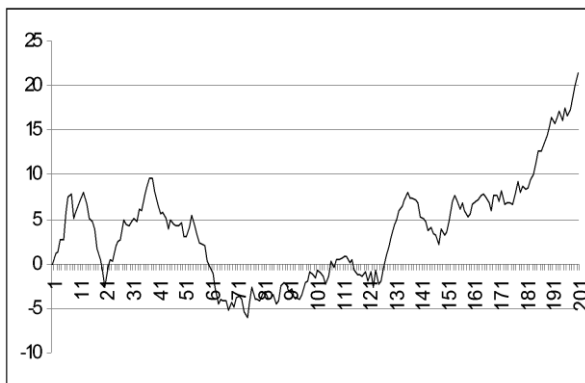
Graphic 3

If AR (p) has a root that is equal to the unity, then the series also has a unit root and has a stochastic trend.

So is concluded that the random walk model presents variations in mean and variance that increase with time, and can change with or without ceasing to be a non-stationary stochastic process.

On the other hand, if the trend in the time series is predictable and not variable is deterministic and if is not predictable is stochastic.

Empirical model to evaluate CPI futures contracts in June 2011 series

In recent years, the volume of trading on stock indexes such as the CPI has shown exponential growth levels, ranking among the futures contracts major operation in the Mexican derivatives market (MexDer), so the analysis of the series they operate, it is important to verify the assumption that data can be modeled under the assumption of normality of geometric Brownian motion.

Considering a historical record of the prices of the underlying asset with as significant differences in expected returns. Be part of the properties of the log-normal model applied to the price action of a futures contract on the CPI, and based on a stochastic process called geometric Brownian motion, this section will implement the theoretical concepts developed in the theoretical framework to model empirically the uncertainties that tracks the price of the underlying asset over time, and determine whether a finite number of data underestimates or overestimates its value is explained by relating the model variables that govern their behavior.

Stochastic methodology for assessing the behavior of prices in the CPI

As described Venegas (2007), as in the case of a share certificate, is often assumed that the percentage change in a stock market index, St, is a differential equation of the type:

$$\frac{ds_t}{s_t} = \mu d_t + \sigma dw_t \qquad (44)$$

Where:

$\frac{ds_t}{s_t}$ - Random variable of the underlying asset.

μd_t - Expected return.

σdw_t - Price volatility of the underlying asset.

To model the variables that make up the model under the assumption that the behavior pattern of price uncertainty corresponds to a probability distribution log-normal considering the historical record of the daily prices of IPC information corresponding to the second quarter 2011, which are sufficient to estimate the behavior of the underlying asset over time and confirm to display the stochastic behavior.

By taking positions in the market, it is important to know what factors determine the asset prices that are quoted in it and if they are directly related to the change.

The objective is to evaluate the behavior of price levels, for which a log-normal model valuation in which the endogenous variable is used as closing prices of the underlying asset and as an exogenous variable uncertainty underlying asset price raises in the future.

From a practical standpoint, if S (t) is given by:

$$S(t) = f\left(t, B(t)\right) = S(0) \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B\right)(t)) \qquad (45)$$

To: t > 0.

Equation (45) aims to capture the behavior of the time path of the underlying asset, for which one must first calibrate the parameters μ and σ; for this is better suited to handle (45) in the equivalent form is derived by taking logarithms:

$$\ln\left(S(t)\right) - \ln\left(S(0)\right) = \left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B\,(t) \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)t; \sqrt{\sigma}t\right), \qquad (46)$$

Where the statistical distribution is a linear transformation $B(t) \sim N\left(0; \sqrt{t}\right)$.

Now considered a collection of quotations k +1 of the active, then:

$$S(0), S(\Delta t) = S_1, S(2\Delta t) = S_2, ..., S(k\Delta t) = Sk, \qquad (47)$$

In the moments 0, Dt, 2Δt, ..., t = kΔt equally spaced in the interval [0, t].

In each sub-period [(j-1) At, jΔt] with 1 $\leq$ j $\leq$ j $\leq$ k we consider the k increases:

$$\mu_j = \ln\left(S(j\Delta_t)\right) - \ln\left(S(j-1\Delta t)\right), 1 \leq j \leq k. \qquad (48)$$

It is observed that for (46) is obtained:

$$\ln\left(S(j\Delta t)\right) = \ln\left(S(0)\right) + \left(\mu - \frac{\sigma^2}{2}\right)(j\Delta t) + \left(\sigma B(j\Delta t)\right) \qquad (49)$$

$$\ln\left(S((j-1)\Delta t)\right) = \ln\left(S(0)\right) + \left(\mu - \frac{\sigma^2}{2}\right)((j-1)\Delta t) + \sigma B((j-1)\Delta t) \qquad (50)$$

Where highlighting (48) of (49) can be expressed (50) in the form:

$$\mu_{j=} \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma(Bj\Delta t) - B\big((j-1)\Delta t\big)), \qquad (51)$$

Now by definition as geometric Brownian motion B (t) has independent increments complies with zero mean and variance t, to increase the length of t, we have:

$$B(_j\Delta_t) - B\big((_j - \Delta t)\big) \sim N(0; \sqrt{\Delta t}), \ 1 \leq_j \leq_j \leq k, \qquad (52)$$

As the random variables μj are independent with mean $\left(\mu - \frac{\sigma^2}{2}\right)\Delta t$ and variance $\sigma^2 \Delta t$. From a sample of k +1 real quote data, you can build the k differences given in (48) and to estimate the parameters μ and σ by the method of moments, which is equal the mean and variance k of the random sample mean μ and the quasi-variables S2 sample variance given by:

$$\mu = \sum_{j=1}^{k}\mu_j \ , \quad S^2 = \frac{1}{k-1}\sum_{j=1}^{k}\mu_j\,(\mu_j - \mu)^2, \qquad (53)$$

This allows building the following system of algebraic equations to estimate the parameters μ and σ:

$$\mu = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t \ , \quad S^2 = \sigma^2\Delta t, \qquad (54)$$

Whose solution is the sought estimates

$$\mu = \frac{1}{\Delta t}\left(\mu + \frac{S^2}{2}\right) \ , \quad \hat{\sigma} = \frac{S}{\sqrt{\Delta t}} \qquad (55)$$

In practice to calibrate the model on daily stock market quotes t = 1/365 is taken when the asset is sensitive to the happenings and events that occur during every day of the year, and for this analysis to t = 1/250, when the underlying asset price only depends on the decisions made by investors in trading hours for 250 working days a year.

In order to apply the model (44) is necessary to simulate B (t) ~ N (0, √ t) through the simulation of log-normal model.

Starts in the model (44) and once calibrated the parameters are used in the following expression:

$$\hat{S}(t) = S(0)\exp\left(\left(\hat{u} - \frac{\hat{\sigma}^2}{2}\right)t + \hat{\sigma}B(t)\right) \qquad (56)$$

It takes into account that B (t) = √ (tZ) with Z ~ N (0, 1), and the model is obtained:

$$\hat{S}(t) = S(0)\exp\left(\left(\hat{u} - \frac{\hat{\sigma}^2}{2}\right)t + \hat{\sigma}\sqrt{t}Z\right) \qquad (57)$$

So generating different values of Z will have different estimates point Ŝ (t) for the value of the asset at time t.

Is expected that the values throwed by the model are higher or equal to the market price, so can be sense a tendency to shed equal or greater value relative to market value, which can be interpreted as an overvaluation of the value of contract, because the model considered as a variable to determine the value, the total value of the contract, and the uncertainty that defines when the risk is higher.

Data Description

The data set used to form the test sample is taken from the operations registers of MexDer for future contracts JN11 CPI series from the historical database of the first quarter of 2011 with an expiry date of 17 June, in order to model the impact of changes in the closing price of the daily operations of the market.

Registered Operations of the CPI futures contracts for the second quarter of 2011

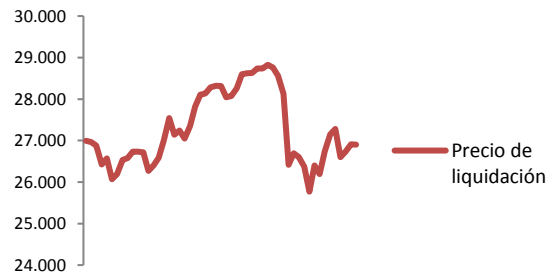| Fecha de operación	Clase	Serie	Núm. de Operaciones	Precio de liquidación	Fecha de operación	Clase	Serie	Núm. de Operaciones	Precio de liquidación
01/04/2011	IPC	JN11	51	26,995	13/05/2011	IPC	JN11	121	28,040
04/04/2011	IPC	JN11	116	26,963	16/05/2011	IPC	JN11	101	28,076
06/04/2011	IPC	JN11	112	26,876	17/05/2011	IPC	JN11	180	28,256
07/04/2011	IPC	JN11	103	26,425	18/05/2011	IPC	JN11	140	28,600
08/04/2011	IPC	JN11	89	26,572	19/05/2011	IPC	JN11	130	28,618
11/04/2011	IPC	JN11	190	26,070	20/05/2011	IPC	JN11	101	28,630
12/04/2011	IPC	JN11	124	26,194	23/05/2011	IPC	JN11	13	28,733
13/04/2011	IPC	JN11	207	26,533	24/05/2011	IPC	JN11	110	28,736
14/04/2011	IPC	JN11	141	26,581	25/05/2011	IPC	JN11	103	28,825
15/04/2011	IPC	JN11	26	26,735	26/05/2011	IPC	JN11	150	28,761
18/04/2011	IPC	JN11	144	26,732	27/05/2011	IPC	JN11	89	28,560
19/04/2011	IPC	JN11	152	26,720	30/05/2011	IPC	JN11	154	28,125
20/04/2011	IPC	JN11	125	26,273	31/05/2011	IPC	JN11	243	26,419
25/04/2011	IPC	JN11	125	26,400	01/06/2011	IPC	JN11	206	26,693
26/04/2011	IPC	JN11	107	26,589	02/06/2011	IPC	JN11	209	26,603
27/04/2011	IPC	JN11	142	27,000	03/06/2011	IPC	JN11	129	26,370
28/04/2011	IPC	JN11	126	27,543	06/06/2011	IPC	JN11	124	25,770
29/04/2011	IPC	JN11	190	27,140	07/06/2011	IPC	JN11	104	26,400
02/05/2011	IPC	JN11	156	27,240	08/06/2011	IPC	JN11	66	26,195
03/05/2011	IPC	JN11	87	27,050	09/06/2011	IPC	JN11	88	26,743
04/05/2011	IPC	JN11	162	27,352	10/06/2011	IPC	JN11	70	27,149
05/05/2011	IPC	JN11	142	27,822	13/06/2011	IPC	JN11	70	27,277
06/05/2011	IPC	JN11	119	28,102	14/06/2011	IPC	JN11	40	26,600
09/05/2011	IPC	JN11	87	28,138	15/06/2011	IPC	JN11	74	26,733
10/05/2011	IPC	JN11	101	28,279	16/06/2011	IPC	JN11	60	26,913
11/05/2011	IPC	JN11	111	28,319	17/06/2011	IPC	JN11	28	26,901
12/05/2011	IPC	JN11	121	28,310					

Table 1

Data values daily operation of CPI futures contracts, taken at the end of the sessions in the MexDer from 1 April to 17 June 2011 are presented in Table 1.

The first column contains the date of operation that were traded the futures contracts; the second column contains the type of underlying asset traded in the third integrated suite operated by the first letter and second consonant of the expiration month and the last two digits of the year of maturity, the fourth column is the number of operations carried out by contract by the stipulated date and finally the settlement price for each contract.

Graph 4 shows the variations of the daily prices on future contracts with the value of the CPI due date to June 17, 2011; on the ordinate is presented the point value of the CPI values governing the contracts at the end of each session in the MexDer, reflecting the market price and at first glance the only worth thing is its erratic up and down behavior that seems to respond to any pattern or mathematical law.

uarterly behavior of CPI futures contracts on MexDer

ISSN-Print: 2007-1582|- ISSN-On line: 2007-3682
ECORFAN®All rights reserved.

Ortega M. Galicia A. Stochastic valuation of futures contracts of IPC in the Mexican derivatives market. ECORFAN Journal-Mexico 2011, 2-5: 380-394

Results

In order to calculate the value of futures contracts on the CPI, the geometric Brownian motion model was used by Monte Carlo simulation to estimate the value of IPC corresponding to the underlying asset that was used in the futures contracts, by comparison between the actual data reported by the MexDer market values calculated in empirically undervaluation or overvaluation way having the use of the model from those selected for the calculation that is the fundamental basis of the data analysis, the process was repeated to find the best result.

To determine the value of the underlying mean values were used for the contract, the geometric Brownian model of value and a random walk model was applied to make a more specific analysis of the variation in price estimates as shown in Table 2 that as follows:

Estimates of the geometric Brownian motion and random walk

Fecha de operación	Precio de mercado	Precio de liquidación al vencimiento	Movimiento geométrico browniano	Caminata aleatoria	Fecha de operación	Precio de mercado	Precio de liquidación al vencimiento	Movimiento geométrico browniano	Caminata aleatoria
01/04/2011	26,448	26663.00			13/05/2011	28,040	27906.89	29336.15	28197.26
04/04/2011	26,995	26664.45	28951.91	26663	16/05/2011	28,076	27972.23	28988.10	27906.89
06/04/2011	26,963	26619.37	28907.66	26664.45	17/05/2011	28,256	28262.65	29009.99	27972.23
07/04/2011	26,876	26566.28	28813.12	26619.37	18/05/2011	28,600	28539.69	29264.80	28262.65
08/04/2011	26,425	26135.6	28710.15	26566.28	19/05/2011	28,618	28498.75	29504.89	28539.69
11/04/2011	26,572	26281.64	28200.01	26135.6	20/05/2011	28,630	28491.07	29415.94	28498.75
12/04/2011	26,070	25783.04	28312.71	26281.64	23/05/2011	28,733	28590.17	29361.47	28491.07
13/04/2011	26,194	25885.8	27731.62	25783.04	24/05/2011	28,736	28589.66	29416.97	28590.17
14/04/2011	26,533	26247.9	27798.08	25885.8	25/05/2011	28,825	28715.96	29369.89	28589.66
15/04/2011	26,581	26324.38	28142.32	26247.9	26/05/2011	28,761	28676.48	29452.96	28715.96
18/04/2011	26,735	26480.03	28179.66	26324.38	27/05/2011	28,560	28505.72	29365.92	28676.48
19/04/2011	26,732	26480.33	28301.42	26480.03	30/05/2011	28,125	28046.16	29144.85	28505.72
20/04/2011	26,720	26558.5	28256.95	26480.33	31/05/2011	26,419	26418.82	28629.61	28046.16
25/04/2011	26,273	26112.87	28295.51	26558.5	01/06/2011	26,693	26638.95	26925.73	26418.82
26/04/2011	26,400	26213.38	27776.71	26112.87	02/06/2011	26,603	26647.65	27107.12	26638.95
27/04/2011	26,589	26432.25	27839.49	26213.38	03/06/2011	26,370	26321.12	27073.06	26647.65
28/04/2011	27,000	26810	28027.51	26432.25	06/06/2011	25,770	25788.37	26699.00	26321.12
29/04/2011	27,543	27338.3	28383.07	26810	07/06/2011	26,400	26355.64	26117.20	25788.37
02/05/2011	27,140	26899.34	28896.57	27338.3	08/06/2011	26,195	26184.39	26649.46	26355.64
03/05/2011	27,240	27045.71	28387.59	26899.34	09/06/2011	26,743	26773.79	26434.40	26184.39
04/05/2011	27,050	26834.05	28496.89	27045.71	10/06/2011	27,149	27106.53	26986.65	26773.79
05/05/2011	27,352	27135.37	28229.12	26834.05	13/06/2011	27,277	27261.17	27278.80	27106.53
06/05/2011	27,822	27561.49	28500.93	27135.37	14/06/2011	26,600	26589.2	27391.00	27261.17
09/05/2011	28,102	27842.76	28902.68	27561.49	15/06/2011	26,733	26719.32	26673.55	26589.2
10/05/2011	28,138	27933.07	29151.43	27842.76	16/06/2011	26,913	26883.53	26761.67	26719.32
11/05/2011	28,279	28067.4	29199.70	27933.07	17/06/2011	26,901	26901.42	26920.81	26883.53
12/05/2011	28,319	28123.76	29293.69	28067.4					

Table 2

Knowing the actual price of June 17, 2011, and compared with the prediction obtained by the geometric Brownian motion model by a difference of 0.19 points between the actual value of each contract is 269.01, and the prediction is 269.20, predicting these values are remarkable.
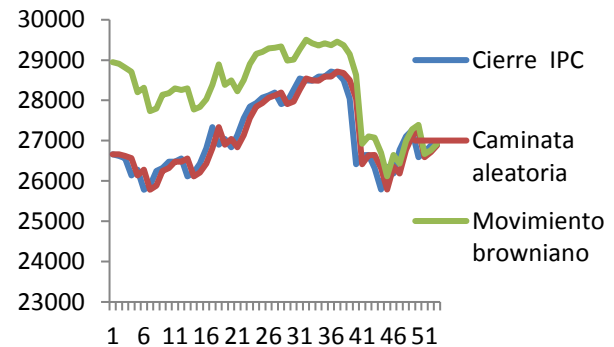
However, this comparative analysis, a random walk model was included to make another estimate obtaining a difference of 0.17 points between the actual value and the random walk of 268.83, so in principle is concluded that the model of geometric Brownian motion is acceptable to predict the price of the underlying asset in a post-day series of data used.

Brownian simulation model prices.

In Graph 5, it is appreciated that the values produced by the model follow the same trend through the day with the assumption that the behavior of prices follow a log-normal distribution is checked, indicating that the geometric Brownian movement correctly evaluated the behavior of the underlying asset price; the significant results of the estimates of the prices shown.

Because arriving expiration date of the contract value and the market of the future converge at the same point.

Ortega M. Galicia A. Stochastic valuation of futures contracts of IPC in the Mexican derivatives market. ECORFAN Journal-Mexico 2011, 2-5: 380-394

Applying the model of geometric Brownian motion price evaluation of the futures contracts in the period studied shows that effectively reflects the uncertainties to which the underlying asset is subject to a stochastic process, increasing investment preferences entrepreneurs.

In annualized cases, the determination of future price using the geometric Brownian motion model with the selected parameters and the information one year of history, shows a behavior that results overvalued price determined by the market.

Conclusions

In the present investigation, it is show that variations of the daily futures prices on the CPI value maturing at June 17, 2011, have an erratic rise and fall that serves no pattern or mathematical law, therefore, the model was more realistic about the behavior of prices of financial assets, is the geometric Brownian motion, which was applied to the valuation of futures contracts on CPI that in the period under review is considered appropriate for the valuation process.

Despite the performance analysis of the assets of the derivatives market shows a complex and erratic behavior, which is explained by the empirical evidence that supports the proposed hypothesis, is assumed that if the geometric Brownian motion describes the behavior of the value, the underlying asset and predicts future price with adequate precision as denote the results, then trading in a futures contract will be made at a fair price.

For the seller and the buyer from the economic point of view which reduces the possibility of arbitration.

⊠

The stochastic model tends to shed normally distributed yield values for calculated parameters that shows an overestimation of the value of contracts that are traded on the MexDer, comparison of the results of the diffusion of the contract price for the future, through a random walk denotes a greater error compared with actual data and this is because the model does not absorb market trend.

The results obtained through the retrospective analysis of historical data and the simulation of stochastic evolution of price of futures contracts CPI corroborated convergence occurring between the price of the futures contract and the market price the underlying asset, in all cases analyzed was found that as the expiration date of the contract the difference between the two prices becomes less about it, confirming that the derivatives market is a good price maker.

The results of the calculation to determine the price of the IPC futures contracts, in the comparison between the actual price of the June 17, 2011 and the results of 53 simulations of the price, geometric Brownian motion, denotes the characteristic of stochastic behavior the price diffusion process, the result obtained was a difference of 0.19 points series between the actual value of each contract is 269.01 and 269.20 of predicting which means that the estimate is close to reality, and represents a suitable alternative for determining the price of futures contracts.

Regardless of the path that describes the price; Volatility is dependent on the length of the gap which is involved in the calculation process and the other side of the observation period which is used for determining the model parameters.

One point to note is that the model of geometric Brownian motion is a simple tool for calculating and analyzing the expectations of the stochastic behavior of prices for futures contracts, in addition to its numerical resolution is achieved through IT applications ensuring the solution and the process time.

It is evident the usefulness of financial assets may have to make an investment; as well as reduce risk and generate returns, attend an important development of the derivatives market in Mexico, in addition to generating models of simple and coherent assessment that allow operators to increase the gain approaching an accurate estimate of the premium risk. However, the valuation method in simple futures contracts can clearly show the stochastic behavior of the price of futures contracts whose volatility depends on the conditions of the economic environment, it is possible to continue the research by incorporating extreme variations or abrupt jumps in prices arising from financial market imperfections.

References

AMIB A.C., (1994), *Operación del mercado de valores en México*, material de apoyo, Bolsa mexicana de valores, 274 paginas. México.

AMIB A.C., (1996), *Mercado de derivados*, material de apoyo, Bolsa mexicana de valores, 142 paginas. México.

Brealey Myers, Marcus, (1997), *Fundamentos de finanzas corporativas*, McGraw Hill, 1ª. Ed, México.

Burton, Malkiel, (2005), *Un paseo aleatorio por Wall Street*, Prentice Hall, México.

Cortina, Ortega Gonzalo, (1995), *Prontuario bursátil y financiero*, Trillas, México.

Cox y Rubinstein, (1979), *Option markets*, Prentice Hall, 7a Ed., U.S.A.

De Alba, Monroy Arturo, (2000), *El mercado de dinero y capitales y el sistema financiero mexicano*, Pac S.A. de C.V., México.

De Lara Haro, Alfonso, (2002), *Medición y control de riesgos financieros*, Limusa, Noriega Editores, México.

Díaz, Carmen, (1998), *Futuros y opciones, teoría y práctica*, Prentice Hall, 1ª. Ed., México.

Díaz Tinoco, Jaime y Trillo Hernández, Fausto, (2002), *Futuros y opciones financieras*, una introducción, Limusa Noriega Editores, México.

Diez de Castro, Mascareñas J.,(2001), *Ingeniería financiera*, McGraw-Hill, México.

Fabozzi, Modigliani, (1998), *Mercados e instituciones financieras*, Prentice Hall, 1ª. Ed., México.

García Santillán, A(2007), *Sistema financiero mexicano y el mercado de derivados*, Edición electrónica gratuita.

Gitman J, Lawrence, (2000), *Principios de Administración financiera*, Pearson, 8ava. Ed., México.

Grandio, Antoni, (1999), *Mercados financieros*, Mc-Graw Hill, 1ª. Ed., México.
Hanke, John E. y Reitsch, Arthur G., (2005), *Pronósticos en los negocios*, Prentice Hall, U.S.A.

Hull, John C, (2006), *Introducción a los mercados de futuros y opciones*, Prentice Hall, 6ª Ed., México.

Jorion, Philippe, (2003). *Valor en riesgo*: El nuevo paradigma para el control de riesgos con derivados, Limusa, Noriega Editores, México.

Jorion, Philippe, (1999), *Value at risk, the new benchmark for controlling   derivatives risk*, McGraw Hill, U.S.A.

Martínez Venegas, Francisco, (2006), *Riesgos financieros y económicos*, Thomson, 1ª. Ed., México.

Pinilla, Roberto: Valero, R. Luís y Guzmán V, Alexander, (1999), *Operaciones financieras en el mercado bursátil*, McGraw Hill, Colombia.

Sánchez Tabales, Ramón, (2004), *La innovación financiera en los mercados internacionales*, FCE, México.

Van Horne, James, (1997), *Administración financiera*, Prentice Hall, 9ª Ed., México.